New state of the matter between the Fermi glass and the Wigner crystal in two dimensions

Giuliano Benenti, Xavier Waintal and Jean-Louis Pichard
CEA, Service de Physique de l'Etat Condensé, Centre d'Etudes de Saclay, 91191 Gif-sur-Yvette, France

Spinless fermions with Coulomb interaction in a square disordered lattice form a new state of the matter which is nor a Fermi glass, neither a Wigner crystal for intermediate Coulomb interaction. From a numerical study of small clusters, we find that this new state occurs between the two critical carrier densities where insulator-metal-insulator transitions of a hole gas have been observed in GaAs.

PACS: 71.30.+h, 72.15.Rn

An important parameter for a system of charged particles is the Coulomb energy to Fermi energy ratio r_s. In a disordered two-dimensional system, the ground state is obvious in two limits. For large r_s, the charges form a pinned Wigner crystal, the Coulomb repulsion being dominant over the kinetic energy and the disorder. For small r_s, the interaction becomes negligible and the ground state is a Fermi glass with localized one electron states, in agreement with the scaling theory of Anderson localization. There is no theory for intermediate r_s, while many transport measurements following the pioneering works of Kravchenko et al [1] and made with electron and hole gases give evidence of an intermediate metallic phase in two dimensions, observed [2] for instance when $6 < r_s < 9$ for a hole gas in GaAs heteostructures. Our study of spinless fermions with Coulomb repulsion in small disordered $2d$ clusters confirms that there is a new ground state for those values of r_s. In a given cluster, as we turn on the interaction, the Fermi ground state can be followed from $r_s = 0$ up to a first level crossing at r_s^F. A second crossing occurs at a larger threshold r_s^W after which the ground state can be followed to the limit $r_s \to \infty$. There is then an intermediate state between r_s^F and r_s^W. In small clusters, the location of the crossings depends on the considered random potentials, but a study over the statistical ensemble of the currents supported by the ground state gives us two well defined values for r_s^F and r_s^W: Mapping the system on a torus threaded by an Aharonov-Bohm flux, we denote respectively I_l and I_t the total longitudinal (direction enclosing the flux) and transverse parts of the driven current. One finds for their typical values $|I_t/I_l| \approx \exp -(r_s/r_s^F)$ and $I_l \approx \exp -(r_s/r_s^W)$ with $r_s^F < r_s^W$. For the Fermi glass, the flux gives rise to a glass of local currents and the sign of I_l can be diamagnetic or paramagnetic, depending on the random potentials. For the intermediate state ($r_s^F < r_s < r_s^W$), the transverse current is suppressed while a plastic flow of longitudinal currents persists up to r_s^W, where charge crystallization occurs. The sign of I_l can be paramagnetic or diamagnetic depending on the filling factor (as for the Wigner crystal), but does not depend on the random potentials (in contrast to the Fermi glass). This suggests that a theorem [3] giving the sign of the current in $1d$ could be extended in $2d$ when $r_s > r_s^F$. For a disorder yielding Anderson localization inside the clusters, one finds $r_s^F \approx 5.8$ and $r_s^W \approx 12.4$ in agreement with the values given by transport measurements which we shortly review.

In exceptionally clean GaAs/AlGaAs heterostructures, an insulator-metal transition (IMT) of a hole gas results [4] from an increase of the hole density induced by a gate. This occurs at $r_s \approx 35$, in close agreement to $r_s^W \approx 37$, where charge crystalization takes place according to Monte Carlo calculations [5], and makes highly plausible that the observed IMT comes from the quantum melting of a pinned Wigner crystal. The values of r_s where an IMT has been previously seen in various systems (Si-Mosfet, Si-Ge, GaAS) are given in Ref. [4], corresponding to different degrees of disorder (measured by the elastic scattering time τ). Those r_s drop quickly from 35 to a constant value $r_s \approx 8 - 10$ when τ becomes smaller. This is again compatible with $r_s^W \approx 7.5$ given by Monte Carlo calculations [6] for a solid-fluid transition in presence of disorder. If the observed IMT are due to interactions, it might be expected that this metallic phase will cease to exist as the carrier density is further increased. This is indeed the case [2] for a hole gas in GaAs heterostructures at $r_s \approx 6$ where an insulating state appears, characteristic of a Fermi glass with weak electron-electron interactions. A similar observation has been also reported [7] for electrons in Si-Mosfet at a lower $r_s \approx 2$. The second transition towards Fermi glass can only be easily seen if the disorder is sufficient for driving localization effects inside the phase breaking length, a condition which weakens the metallic behavior of the intermediate phase.

The purpose of this work is to take advantage of exact diagonalization techniques possible only for small clusters at low filling factors. One can question the ability of this method to give results valid for large systems. In the clean limit, Pikus and Efros [5] have obtained $r_s^W \approx 35$ diagonalizing 6×6 clusters with 6 particles, close to $r_s^W \approx 37$ obtained by Tanatar and Ceperley for the thermodynamic limit. This is a first reason to

study how one goes from the Fermi glass towards the Wigner crystal in small clusters. A second justification will be given a posteriori by the agreement between our numerical results and the experiments. We consider a two–dimensional model of Coulomb interacting spinless fermions in a random potential. It is defined on a square lattice with L^2 sites occupied by N electrons. The Hamiltonians reads

$$H = -t \sum_{<i,j>} c_i^\dagger c_j + \sum_i v_i n_i + \frac{U}{2} \sum_{i \neq j} \frac{1}{r_{ij}} (n_i n_j), \qquad (1)$$

where $c_i^\dagger$ (c_i) creates (destroys) an electron in the site $i = (i_x, i_y)$, $\sum_{<i,j>}$ denotes a sum restrained to nearest neighbors, t is the strength of the hopping terms characterizing the particle kinetic energy ($t = 1$ in the following) and r_{ij} is the inter-particle distance for a $2d$ torus (minimum image convention). The random potential v_i of the site i with occupation number $n_i = c_i^\dagger c_i$ is taken from a box distribution of width W. The strength U of the true physical long-ranged electron-electron interaction, suitable for insulating phases where screening breaks down, gives a Coulomb energy to Fermi energy ratio $r_s = U/(2t\sqrt{\pi\nu})$ for a filling factor $\nu = N/L^2$. For $U = 0$ the Hamiltonian (1) reduces to the Anderson model of localization, while for $t = 0$ it reduces to the Coulomb glass model, which describes classical point charges in a random potential. Exact diagonalization techniques for large sparse matrices (Lanczos method) are used to study a statistical ensemble of small clusters. The boundary conditions are always taken periodic in the transverse y-direction, and are chosen periodic, antiperiodic, or such that the system becomes a torus enclosing an Aharonov-Bohm flux ϕ in the longitunal x-direction.

Fig. 1 exhibits behaviors characteristic of individual small clusters ($L = 6$, $N = 4$), where disorder ($W = 15$) yields a strong Anderson localization, as a function of r_s ($\nu = 1/9$ and $0 < U < 50$). Looking at the low energy part of the spectrum, one can see that, as we gradually turn on the interaction, classification of the levels remains invariant up to first avoided crossings, visible for the ground state at r_s^F, where a Landau theory of the Fermi glass is certainly no longer possible. Looking at the electronic density $\rho_i = \langle \Psi_0 | n_i | \Psi_0 \rangle$ of the ground state $|\Psi_0\rangle$, we have noticed that it is mainly maximum in the minima of the site potentials for the Fermi glass, with restrictions coming from kinetic terms and Pauli principle. After the second avoided crossing at r_s^W, ρ_i is negliglible except for four sites forming a lattice of charges as close as possible to the Wigner crystal triangular network in the imposed square lattice. The degeneracy of the crystal is removed by the disorder, the array being pinned in 4 sites of favorable energies. The ρ_i of the intermediate ground state give the impression of a mixed phase with crytalline domains co-existing with glassy domains where fermions occupy the sites of lowest energies.

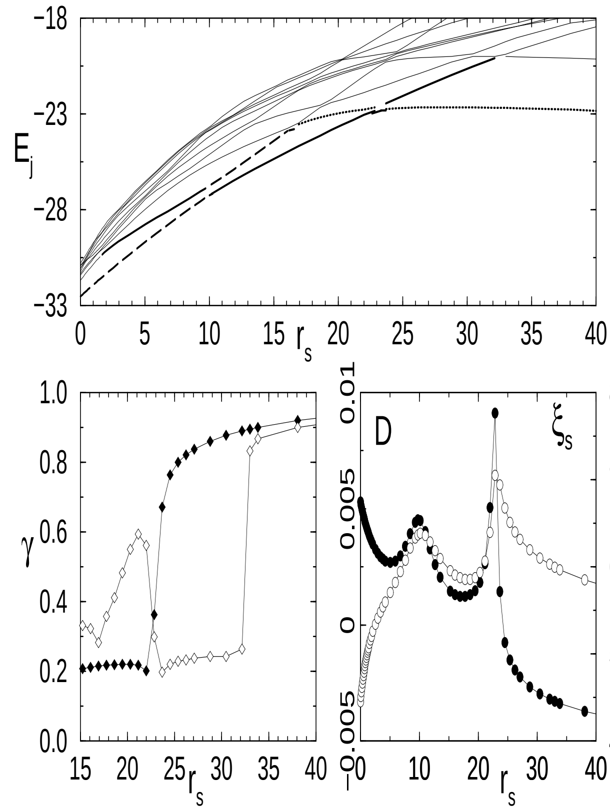

FIG. 1. Top: Low energy spectrum of a small single cluster ($N = 4$, $L = 6$, $W = 15$) (a $1.9r_s$ term has been substracted); Bottom left: jumps of γ at the second crossing where the ground state (filled diamonds) and the first excited state (empty diamonds) are interchanged. Bottom right: Ground state sensitivity D (left scale, empty circles) and number of occupied sites ξ_s (rigth scale, filled circles).

For the same cluster, we have calculated the density-density correlation function $C(r) = N^{-1} \sum_i \rho_i \rho_{i-r}$ and the parameter γ used by Pikus and Efros [5] for characterizing the melting of the crystal. The parameter $\gamma = \max_r C(r) - \min_r C(r)$ calculated for the ground state and the first excited state around r_s^W allows us to identify the second crossing with the melting of the crystal, since $\gamma = 1$ for a crystal and 0 for a liquid. Moreover, one can see that the Wigner crystal becomes unstable in the intermediate phase, while the ground state is related to the first excitation of the Wigner crystal above r_s^W (Fig. 1 bottom left). The general picture is reminiscent to those found [8] in one dimensional models. As we increase r_s, there are level crossings associated to charge reorganizations of the ground state. As in ref. [8], those charge reorganizations are accompanied by noticeable delocalization effects of the ground state (Fig. 1 bottom right). This is shown by a sharp enhancement of the ground state sensitivity $D = E(0) - E(\pi)$ measuring the change of the ground state energy when the boundary conditions are twisted in the x-direction, i.e. more fundamentally the ability of the ground state to a support

a persistent current when the system forms a torus enclosing a flux. The second quantity demonstrating the delocalization effect at the crossings is the participation ratio $\xi_s = N^2(\sum_i \rho_i^2)^{-1}$ of the ground state, i.e. the number of sites that it occupies. Fig.1 is representative of the ensemble, with the restriction that the location of the crossings fluctuates from one sample to another, as observed [8] in $1d$, as well as the sign (paramagnetic or diamagnetic) of D below r_s^F, in contrast to $1d$.

Keeping the boundary condition periodic in the transverse y-direction and such that the system encloses a flux ϕ (in dimensionless units) in the longitudinal x-direction ($\phi = \pi$ coresponds to anti-periodic condition), one drives a persistent current of total longitudinal and transverse components given by:

$$I_l = -\frac{\partial E(\phi)}{\partial \phi} = \frac{\sum_i I_i^x}{L_y} \text{ and } I_t = \frac{\sum_i I_i^y}{L_x}$$

respectively. The local current flowing at the site i in the x-direction is defined by

$$I_i^x = 2\mathrm{Im}(\langle \Psi_0 | c^\dagger_{i_x+1,i_y} c_{i_x,i_y} | \Psi_0 \rangle)$$

and by a corresponding expression for I_i^y. The response is paramagnetic if $I_l > 0$ and diamagnetic if $I_l < 0$. Legett theorem [3], based on a simple variational possiblity for the ground state wave function Ψ_0, states that the sign of the response is given by the parity of N. $(-1)^N(E(0) - E(\pi))$ is always positive in $1d$ for all disorder and interaction strength. The proof is based on the nature of "non symmetry dictated nodal surfaces ", which is trivial in $1d$, but which has a quite complicated topology in higher d. Moreover, in $2d$ and small r_s, the sign of I_l depends on the site potentials.

We have separately studied the paramagnetic and diamagnetic samples of an ensemble of 10^3 clusters with again $L = 6$, $N = 4$ and $W = 15$. We obtain lognormal distributions for all values of r_s for a large disorder ($W = 15$), as illustrated by Fig.2. The dependence on r_s of the averages and variances of the logarithms of the paramagnetic $D_+ = E(0) - E(\pi) > 0$ and diamagnetic $D_- = E(0) - E(\pi) < 0$ responses are given in Fig.3. The log-averages exponentially decay as $D_- \propto \exp -(r_s/r_s^F)$ and $D_+ \propto \exp -(r_s/r_s^W)$ with $r_s^F \approx 5.8$ and $r_s^W \approx 12.4$. The decay of the typical values are accompanied by an increase of their fluctuations. The r_s-dependence of the log-variances are shown in the insert, showing that the variances and the averages are proportional. This reminds us a similar relation [9] for the log-conductance in a $2d$ Fermi glass at $r_s = 0$. The variances of $\log |I_t/I_l|$ and $\log D_+$ increase as r_s/r_s^F, and as r_s/r_s^W above r_s^W respectively. Both the average and the variance of $\log |D_+|$ remain nearly constant in the intermediate phase.

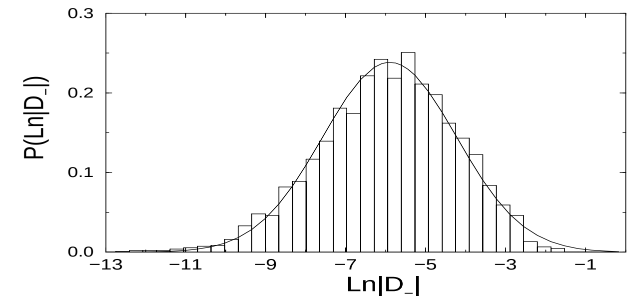

FIG. 2. Distribution of the ground state sensitivity D_- of the diamagnetic samples of an ensemble of 10^4 clusters at $r_s = 1.7$ fitted by a log-normal.

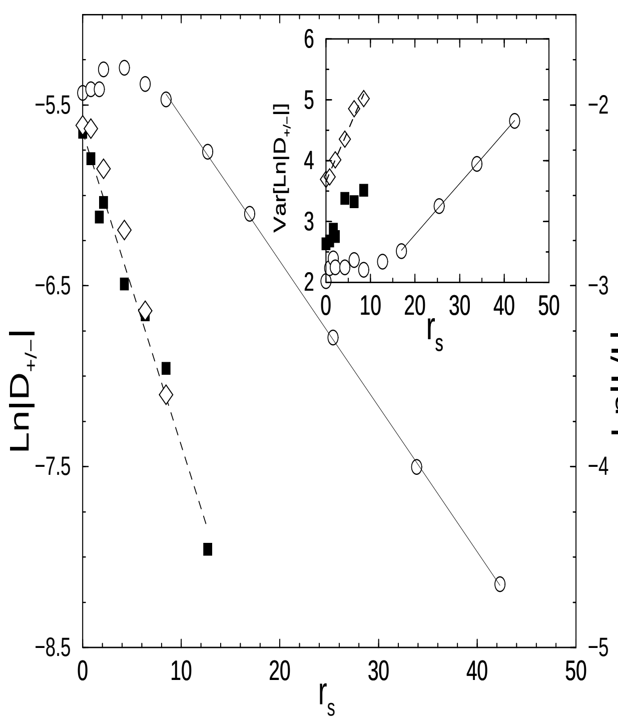

FIG. 3. Log-averages of $D_+ \propto \exp -(r_s/12.4)$ (paramagnetic, empty circle) and $|D_-| \propto \exp -(r_s/5.8)$ (diamagnetic, filled square) given by the left scale, and of the ratio $|I_t/I_l|$ (empty diamond, right scale). Insert: variances of $\log |D_-|$, of $\log |I_t/I_l| \propto r_s/5.9$ and of $\log |D_+| \propto r_s/11.9$.

A transition from glassy towards plastic flow as r_s increases has been found [10] by Berkovits and Avishai. It is likely that the existence of diamagnetic samples comes from disordered arrays of local loops of current which should give rise to a transverse current I_t. Studying the distribution of I_t and I_l at $\phi = \pi/2$, we find that there is indeed a non zero transverse current I_t when r_s is small. Its sign is random over the statistical ensemble, and the ratio $\log |I_t/I_l|$ is normally distributed. The log-average plotted in Fig.3 confirms that $|I_t/I_l| \propto \exp -(r_s/r_s^F)$ exactly as D_-.

The fact that I_l becomes paramagnetic independently on the microscopic disorder when $r_s > r_s^F$ does not mean

that Coulomb repulsions always yield this response. For instance, 4×6 clusters with $N = 6$ particles become always diamagnetic when r_s increases. One just concludes that there is a rule for $r_s > r_s^F$ giving the sign of the response in $2d$, as the one found by Legett in $1d$, which does not depend on the random potential.

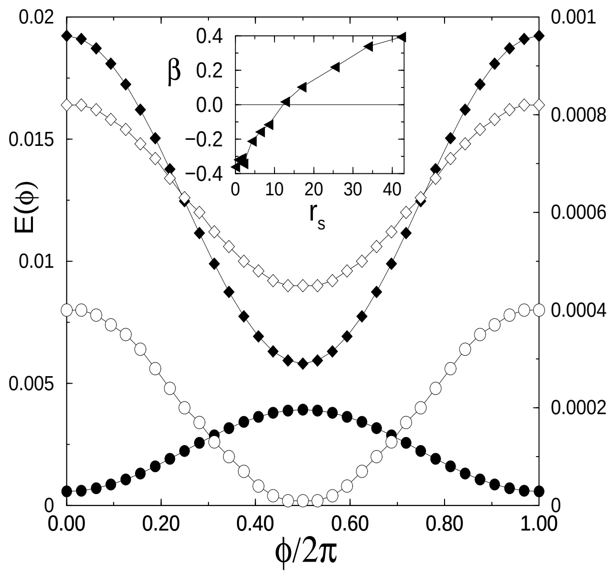

FIG. 4. Flux dependence of ground state and first excited state energies for $r_s = 0$ (filled symbols, left scale) and $r_s = 42$ (empty symbols, right scale) for a single cluster. A suitable energy translation has been done to put the curves in the same figure. Insert: ensemble average of $\beta = D_0 D_1 / |D_0 D_1|$.

It is also interesting to study the currents supported by the first excited state. In a Fermi glass, if the ground state current $I_l(E_0)$ has a certain sign, the current $I_l(E_1)$ carried by the first excitation has very often the opposite sign. We have noticed that the opposite behavior becomes more likely when r_s increases: the currents carried by the ground state and the first excited state flow in the same direction. An illustration is given in Fig. 4, where the flux dependence of the two first levels at $r_s = 0$ and $r_s = 42$ of the same cluster is shown. The transition from anti-correlated towards correlated flux dependence can be seen on the average of $D_0 D_1 / |D_0 D_1|$ which precisely changes its sign at r_s^W, as shown in the insert. Fig.4 also confirms that the approximation $-\partial E_j / \partial \phi \approx D_j = E_j(0) - E_j(\pi)$ makes sense.

In summary, we have shown that a simple model of spinless fermions with Coulomb repulsion in a random potential can account for the two critical densities of holes where insulator-metal-insulator transitions occur. This gives an important confirmation that the intermediate phase observed in two dimensions results from Coulomb long-range repulsions, and makes unlikely scenarios based on spin-orbit scattering [11] or on some special single electron interface properties [12]. In contrast to Finkelstein's approach [13], we underline that the spins are neglected in our model, and we cannot confirm that the intermediate ground state is a metal for a disorder energy to kinetic energy ratio as large as $W/t = 15$. We have mostly seen noticeable delocalization effects near the crossing points. We cannot exclude that weaker disorders or spin effects are necessary for having a metal. We also note that the possibility of a superconducting state has been discussed [14], and not entirely ruled out in the clean limit [15]. A study of the temperature dependence of the ground state sensitivity and of the system size dependence of the gap excitations at a fixed r_s will be useful for clarifying this issue. From these small cluster studies, we conclude that there is a new state of the matter, clearly separated from the Fermi glass and from the Wigner crystal, identified by a plastic flow of currents and a magnetic response with a sign independent on the microscopic disorder. The obtained critical r_s factors are surprisingly close to those given by Ref. [2]. However, the critical r_s can depend on W, as already known for r_s^W, and might have small finite size corrections (a study of D_- in 8×8 clusters with $N = 4$ gives $r_s^F \approx 6.2$ instead of 5.8).

A partial support by the TMR network "Phase coherent dynamics of hybrid nanostructures" of the EU is gratefully acknowledged.

[1] S.V. Kravchenko et al. *Phys. Rev.* B **50**, 8039 (1994); ibid **51**, 7038 (1995).
[2] A.R. Hamilton, M.Y. Simmons, M. Pepper, E.H. Linfield, P.D. Rose and D.A. Ritchie, preprint cond-mat/9808108.
[3] A.J. Legett, *Granular Nanoelectronics*, D.K. Ferry ed., p 297, Plenum Press, N.Y. (1991).
[4] J. Yoon, C.C. Li, D. Shahar, D.C. Tsui and M. Shayegan, preprint cond-mat/9807235.
[5] B. Tanatar and D.M. Ceperley, *Phys. Rev.* B **39**, 5005 (1989); F.G. Pikus and A.L. Efros, *Solid State Commun.* **92**, 485 (1994).
[6] S.T. Chui and B. Tanatar, *Phys. Rev. Lett.* **74**, 458 (1995).
[7] V.M. Pudalov, G. Brunthaler, A. Prinz and G. Bauer, preprint cond-mat/9812183.
[8] P. Schmitteckert, R.A. Jalabert, D. Weinmann and J.-L. Pichard, *Phys. Rev. Lett.* **81**, 2308 (1998).
[9] S. Feng and J.L. Pichard, *Phys. Rev. Lett.* **67**, 753 (1991).
[10] R. Berkovits and Y. Avishai, *Phys. Rev.* B **57**, R15076 (1998).
[11] V.M. Pudalov, *JETP Lett.* **66**, 175 (1997).
[12] B.L. Altshuler and D.L. Maslov, *Phys. Rev. Lett.* in press.
[13] A.M. Finkelstein, *Sov. Sci. Rev* A *Phys* **14**, 3 (1990).
[14] P. Phillipps et al., *letters to nature* **395**, 254 (1998).
[15] E.V. Tsiper and A.L. Efros, *Phys. Rev.* B **57**, 6949 (1998).